Homework 4. Linear Algebra. Spring 2022. Prof. Pineiro

Print Name: ______________________________

1. Consider the linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ given by

$$T(x) = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 5 & 4 \end{pmatrix} x$$

a. Determine if T is one-to-one.
b. Determine if T is onto.

2. Let V be the vector space of 2×2-matrices. Recall that the trace of a matrix, written $\mathrm{Tr}(A)$, is the sum of its diagonal entries:

$$\mathrm{Tr}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d.$$

a. Prove that the subset S of V consisting of matrices of trace 0 is a subspace of V.
b. Find a basis for S.
c. What is the dimension of S?

3. Consider the linear map $f\colon V \longrightarrow V'$ between the finite dimensional vector spaces V and V'.
a. Show that if $\dim(V) > \dim(V')$ the map **cannot be injective**.
b. Show that if $\dim(V) < \dim(V')$ the map **cannot be surjective**.
c. Find an example of an **injective** linear map $f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^3$.
d. Find an example of a **surjective** linear map $f\colon \mathbb{R}^4 \longrightarrow \mathbb{R}^2$.
d. Find an example of a **bijective** linear map $f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$.
d. Find an example of a **non-bijective** linear map $f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$.

4. Given the matrix $A = \begin{pmatrix} 9 & -4 \\ 12 & -5 \end{pmatrix}$.
a. Find the eigenvalues of A.
b. Find a basis made of eigenvectors.
c. Find a matrix P such that $P^{-1}AP = D$.

5. Given the symmetric matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{pmatrix}$.
a. Find the eigenvalues of A.
b. Find a basis of eigenvectors.
c. Find an orthogonal basis made of eigenvectors.
d. Find an orthogonal matrix P such that $P^{-1}AP = D$.